Stock-dependent discrete-time dynamic pollution games

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Extended abstract

In this paper we address the dynamics of strategic actions in a pollution game model where the payoff matrix of the game depends endogenously on the accumulation or depletion of an environmental stock variable. Our model has the structure of a stock-dependent dynamic game, that is a game where the pay-offs to the players changes with the evolution of an external state variable, which in turn depends on the action of the players. More specifically, we analyse a two-player stock-dependent dynamic game in which the stage game is a normal form game with myopic players who use Markov stationary strategy, that is they play a stage game Nash equilibrium in each period. A dynamic consequence of stock-dependent payoffs on the strategies played is that the qualitative structure of the solution of the stock-dependent stage game might change as the stock variable evolves. Here we develop a method to study these qualitative changes and classify the possible economic outcomes. The method is applied to the discrete time dynamic ecological-economic interest conflicts in the lake pollution game.

The lake game (Brock and de Zeeuw, 2002; M. Scheffer, 1997) is a paradigmatic example of the economic trade-offs in environmental quality: among services provided by a lake, there is also its role as a sink to dampen pollutant substances from agriculture and industries, which contribute to social welfare. Brock and de Zeeuw (2002) propose a repeated lake game where the payoff matrix depends on the history of players actions. Dechert & Donnel (2006) address a stochastic lake. The motivation of this model goes beyond lakes, being a generic representation for ecological systems that can flip in a bad state (Maler et al., 2003): the non-linear dynamics and irreversible nature of ecological systems have huge environmental and economic consequences (Polasky et al., 2011; Scheffer, 2009). What is little understood is how these propagate when the problem is extended in time with repeated interaction between players. The trade-offs in economic activities of exploitation of a lake become a game whenever two or more parties are interested. On an infinite time horizon this is a differential game (Kossioris et al., 2012), the strategic version of an optimal control problem (Moghayer, 2012). In this context we study the lake pollution game as a dynamic stock-dependent game: players payoffs endogenously depend on the accumulated stock of pollution and the Nash equilibria of the stage games change through the history of repeated interaction. Given the classification of symmetric two-player games, we study these qualitative changes in the structure of the solutions of the game and classify the possible economic outcomes.The dynamic of the game, as well as the lake dynamics, entail path-dependence and possibly cyclical dynamics.

Some authors have considered the possibility of a non-constant payoff matrix in the repeated Prisoner Dilemma. In this context, the method that is proposed in this paper could be relevant in particular to environmental problems of the common goods type, where the threat from defection prevents the players from coordinating into a socially desirable equilibrium. The problem of

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emergence of cooperation in the repeated Prisoner Dilemma has been studied extensively in the last two decades (see Nowak, 2006).

A brief description of the model

Consider the following symmetric 2×2 game played by two farmers sharing a lake and doing agricultural activities in the surrounding lands. There is a net flow of pollution (phosphorus)at each period t to the lake, $a^t = {a_1}^t + {a_2}^t$, which originate from the use of two types of the artificial fertilisers that is used by farmers: low-pollution (L) and high pollution (H) where ${a_i}^t \in \{a_H, a_L\}$ and $a_L < a_H$.

	L	H
L	$R(x_t)$,$R(x_t)$	$S(x_t)$,$T(x_t)$
H	$T(x_t)$,$S(x_t)$	$P(x_t)$,$P(x_t)$

Here the payoffs depend on the stock of pollution (concentration of phosphorus in the lake) x_t. The payoff to the player i at time t is defined as

$$\pi_t^i = \log a_t^i - c x_t^2 \in \{R_t, P_t, S_t, T_t\},$$

where

- $\log a_t^i$: benefit to farmer i from using fertilizers
- $c x_t^2$: cost of pollution to each farmer

The lake dynamics is described by

$$x_t = a_t + (1-b)x_{t-1} + \frac{x_{t-1}^q}{1 + x_{t-1}^q}, \quad x_0 \text{ is given,}$$

with

- b: rate of loss of phosphorus due to sedimentation or outflow,
- $\dfrac{x^q}{1+x^q}$: biological production of phosphorus in the lake.

Sketch of the analysis

We first fix the physical parameters b and q, and consider a number of different sets of values for the level of loadings a_L, a_H and the initial state of the lake $x_0 = 0.1$ using the lake response curve (the s-shape locus of the steady state). For each set, by varying the cost parameter c, we analyse the in the pattern of possible changes in the Nash equilibria structure of the stage game played by the farmers at each time period as a result of the change in the payoff rankings, classify the corresponding economic outcomes and environmental impacts. The qualitative changes in the game dynamic is represented geometrically in the payoff space, $R(x) - P(x)$-space, utilising the notion of the bifurcation diagram, the mathematics of classification in the theory of dynamical systems.The diagram is partitioned into twelve different regions corresponding to different types of the two-by-two games (e.g. Prisoner's dilemma, Battle of sexes, Staghunt, Hawk-Dove, ...).

We also vary the *stiffness* or responsiveness of the lake by varying parameter q and analyse its consequences on the dynamic of the game. A strongly responsive lake exhibits more sudden shifts between clean and polluted regimes. In the pollution game of strongly responsive ecosystems, it is more likely that after some initial fluctuation, the long-term strategy of the players is 'green'.